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THE DIEUDONNÉ PROPERTY ON $C(K, E)$

BY

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ABSTRACT. In this paper we prove that if E is a Banach space with separable dual, then the space $C(K, E)$ of all continuous E-valued functions on a compact Hausdorff topological space K has the Dieudonné property.

Introduction. In his important paper [5], Grothendieck axiomatized some relevant properties of $L_1(\mu)$ and $C(K)$ spaces, introducing among others the so-called Dunford-Pettis and Dieudonné properties. After the work of Grothendieck there has been intensive literature on the study of the Dunford-Pettis property in spaces of continuous and integrable vector valued functions, but, as far as we know, little has been written concerning the Dieudonné property. In this paper we prove that if E is a Banach space with separable dual, the space $C(K, E)$ of continuous E-valued functions on a compact Hausdorff topological space K has the Dieudonné property.

Notations and fundamentals. Throughout the paper E and F are Banach spaces, K a compact Hausdorff space, Σ the σ-field of Borel subsets of K, and $C(K, E)$ the Banach space under the supremum norm of the E-valued continuous functions on K.

The notations and terminology used and not defined in this paper can be found in [3, 5 and 7].

If A is a subset of E we denote by $[A]$ the algebraic linear span of A. $L(E, F)$ is the space of all bounded linear operators from E to F. By $E \simeq F$ we mean that there exists an isometric isomorphism between E and F. The Banach space of all regular countably additive measures μ on Σ with values in E and of finite variation on K, endowed with the norm of total variation ($\|\mu\| = |\mu|(K)$), is denoted by rcabv(Σ, E).

It is well known that $C(K, E)^* \simeq$ rcabv(Σ, E^*).

We denote by $H(E)$ the subset of E^{**} formed by all the $\sigma(E^{**}, E^*)$-limits of weakly Cauchy sequences in E.

Recall that a Banach space E is said to have the Dieudonné property if for every Banach space F, a bounded linear operator $T: E \to F$ is weakly compact if and only if T transforms weakly Cauchy sequences into weakly convergent ones.

When E is separable one has the following characterization.

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Proposition [5, p. 161]. *If E is separable then the following two conditions are equivalent*:

(i) *E has the Dieudonné property.*

(ii) *Every $\sigma(E^*, H(E))$-convergent sequence in E^* is weakly convergent.*

To prove the announced result we shall need two lemmas.

Lemma 1. *Let K be metrizable, (A_n) a sequence of pairwise disjoint open subsets of K, and (x_n^{**}) a bounded sequence in E^{**} for which there exists a bounded family of sequences in E, $\{(x_m^n)_m: n \in \mathbf{N}\}$ such that $(x_m^n)_m$ is $\sigma(E^{**}, E^*)$-convergent to x_n^{**} for all $n \in \mathbf{N}$. Then, the element τ of $C(K, E)^{**}$ defined by*

$$\langle \mu, \tau \rangle = \sum_{n=1}^{\infty} \langle \mu(A_n), x_n^{**} \rangle \quad \text{for } \mu \in \operatorname{rcabv}(\Sigma, E^*)$$

belongs to $H(C(K, E))$.

Proof. For every $n \in \mathbf{N}$ let $(K_m^n)_m$ be an increasing sequence of compact subsets of K so that $A_n = \bigcup_m K_m^n$.

For each $n, m \in \mathbf{N}$ take $f_m^n \in C(K)$ such that

$$f_m^n(t) = \begin{cases} 1 & \text{if } t \in K_m^n, \\ 0 & \text{if } t \in K \setminus A_n, \end{cases} \quad \text{and} \quad 0 \leqslant f_m^n(t) \leqslant 1 \quad \text{for all } t \in K.$$

Let us consider the sequence $(\phi_k) \subset C(K, E)$ defined by

$$\phi_k = f_k^1(\cdot)x_k^1 + f_{k-1}^2(\cdot)x_{k-1}^2 + \cdots + f_1^k(\cdot)x_1^k.$$

It is clear that $M = \sup_k \|\phi_k\| < +\infty$. By the definition we have that

(a) if $t \in A_n$ then $\phi_{n+k}(t) = f_{k+1}^n(t)x_{k+1}^n$ for all $k \in \mathbf{N}$, and

(b) if $t \in K_m^n$ then $\phi_{n+k}(t) = x_{k+1}^n$ for all $k \geqslant m$.

Therefore, $(\phi_k(t))$ is weakly Cauchy in E for each $t \in K$. Indeed, if $t \in K \setminus (\bigcup_n A_n)$ then obviously $\lim_k \phi_k(t) = 0$; if $t \in A_n$ then there exists $m \in \mathbf{N}$ so that $t \in K_m^n$ and, by (b), we have that $(\phi_k(t))$ is weakly Cauchy in E. Now, according to Theorem 9 of [4], it follows that (ϕ_k) is weakly Cauchy in $C(K, E)$.

Let $\mu \in \operatorname{rcabv}(\Sigma, E^*)$ and $\varepsilon > 0$. Then there exists $n_0 \in \mathbf{N}$ such that

$$(1) \qquad |\mu|\Big(\bigcup_{n>n_0} A_n\Big) = \sum_{n>n_0} |\mu|(A_n) < \frac{\varepsilon}{8M}.$$

Since $\lim_m |\mu|(A_n \setminus K_m^n) = 0$ for every $n \in \mathbf{N}$, there exists $m_0 \in \mathbf{N}$ such that

$$(2) \qquad |\mu|\big(A_n \setminus K_{m_0}^n\big) < \varepsilon/8Mn_0 \quad \text{for } 1 \leqslant n \leqslant n_0.$$

Because $\lim_m |\langle \mu(K_{m_0}^n), x_n^{**} - x_m^n \rangle| = 0$ for each $n \in \mathbf{N}$, there exists $m_1 \in \mathbf{N}$ so that

$$(3) \qquad \big|\langle \mu\big(K_{m_0}^n\big), x_n^{**} - x_m^n \rangle\big| < \varepsilon/2n_0 \quad \text{for all } m \geqslant m_1 \text{ and } 1 \leqslant n \leqslant n_0.$$

By (1), (2), (3) and (b), if $k \geqslant n_0 + m_0 + m_1$ we have

$$\begin{aligned}|\langle \mu, \tau - \phi_k \rangle| &= \left| \sum_{n=1}^{\infty} \langle \mu(A_n), x_n^{**} \rangle - \int_K \phi_k \, d\mu \right| \\ &\leqslant \sum_{n=1}^{n_0} \left| \left\langle \mu(K_{m_0}^n), x_n^{**} \right\rangle - \int_{K_{m_0}^n} \phi_k \, d\mu \right| + \sum_{n=1}^{n_0} \left| \left\langle \mu\left(\frac{A_n}{K_{m_0}^n} \right), x_n^{**} \right\rangle \right| \\ &\quad + \sum_{n=1}^{n_0} \left| \int_{A_n \setminus K_{m_0}^n} \phi_k \, d\mu \right| + \sum_{n > n_0} |\langle \mu(A_n), x_n^{**} \rangle| + \left| \int_{\cup_{n > n_0} A_n} \phi_k \, d\mu \right| \\ &\leqslant \sum_{n=1}^{n_0} \left| \left\langle \mu(K_{m_0}^n), x_n^{**} - x_{k-n+1}^n \right\rangle \right| + 2M \sum_{n=1}^{n_0} |\mu| \left(\frac{A_n}{K_{m_0}^n} \right) \\ &\quad + 2M \sum_{n > n_0} |\mu|(A_n) < \varepsilon.\end{aligned}$$

And thus we conclude that τ belongs to $H(C(K, E))$.

REMARK. Note that if E does not contain any subspace isomorphic to l_1, then every bounded sequence (x_n^{**}) in E^{**} satisfies the assumption of Lemma 1.

LEMMA 2. *If K is metrizable and (μ_n) is a sequence in* rcabv(Σ, E^*), *$\sigma(C(K, E)^*, H(C(K, E)))$-convergent to zero, then the set $\{ |\mu_n| \colon n \in \mathbf{N} \}$ is uniformly countably additive.*

PROOF. Since $H(C(K, E)) \supset C(K, E)$ the sequence (μ_n) is bounded in rcabv(Σ, E^*). If we suppose that $\{ |\mu_n| \colon n \in \mathbf{N} \}$ is not uniformly countably additive, by VI.2.13 of [3], there exist $\varepsilon > 0$, a sequence (A_n) of pairwise disjoint open subsets of K and a subsequence of (μ_n) (which we still denote by (μ_n)) such that

$$|\mu_n|(A_n) > \varepsilon \quad \text{for all } n \in \mathbf{N}. \tag{4}$$

By using Rosenthal's lemma (see I.4.1 of [3]) it follows that there exists an increasing sequence $(n_j) \subset \mathbf{N}$ so that

$$|\mu_{n_j}| \left(\bigcup_{i \neq j} A_{n_i} \right) < \frac{\varepsilon}{3} \quad \text{for all } j \in \mathbf{N}.$$

It is easily verified that if $\mu \in$ rcabv(Σ, E^*) and A is an open subset of K then $|\mu|(A) = \sup |\Sigma_k \langle x_k, \mu(B_k) \rangle|$, where the supremum is taken over all finite sequences (B_k) of pairwise disjoint open subsets of K contained on A and all finite sequences (x_k) in the unit ball of E. Therefore for each $i \in \mathbf{N}$ there exist a finite sequence $(B_k^i)_{k=1}^{r_i}$ of pairwise disjoint open subsets of K with $\bigcup_{k=1}^{r_i} B_k^i \subset A_{n_i}$ and a finite sequence $(x_k^i)_{k=1}^{r_i}$ in the unit ball of E such that

$$|\mu_{n_i}|(A_{n_i}) < \left| \sum_{k=1}^{r_i} \langle x_k^i, \mu_{n_i}(B_k^i) \rangle \right| + \frac{\varepsilon}{3}.$$

So, by (4), we have

$$\left|\sum_{k=1}^{r_i} \langle x_k^i, \mu_{n_i}(B_k^i)\rangle\right| > \frac{2}{3}\varepsilon.$$

Now we define $\tau \in C(K, E)^{**}$ by

$$\langle \mu, \tau\rangle = \sum_{i=1}^{\infty} \sum_{k=1}^{r_i} \langle x_k^i, \mu(B_k^i)\rangle \quad \text{for } \mu \in \operatorname{rcabv}(\Sigma, E^*).$$

According to Lemma 1 it is clear that $\tau \in H(C(K, E))$.

For every $j \in \mathbf{N}$ we have

$$\left|\langle \mu_{n_j}, \tau\rangle\right| = \left|\sum_{i=1}^{\infty} \sum_{k=1}^{r_i} \langle x_k^i, \mu_{n_j}(B_k^i)\rangle\right|$$

$$\geqslant \left|\sum_{k=1}^{r_j} \langle x_k^j, \mu_{n_j}(B_k^j)\rangle\right| - \sum_{i \neq j} |\mu_{n_j}|(A_{n_i}) > \frac{2}{3}\varepsilon - \frac{\varepsilon}{3} = \frac{\varepsilon}{3}.$$

But this contradicts the assumption that (μ_n) is $\sigma(C(K, E)^*, H(C(K, E)))$-convergent to zero.

THEOREM. *If E is a Banach space with separable dual then $C(K, E)$ has the Dieudonné property.*

PROOF. According to Corollary 8 of [**2**], E is isomorphic to a quotient of a space F which has a shrinking basis. If π: $F \to E$ is a quotient map, there exists a continuous cross-section s: $E \to F$ of π (see, for instance, [**6**, 21.C, Corollary]). So the continuous linear operator that maps $\phi \in C(K, F)$ into $\pi \circ \phi \in C(K, E)$ is onto. In consequence $C(K, E)$ is isomorphic to a quotient of $C(K, F)$ and therefore it is enough to prove the theorem when E has a shrinking basis.

Let (e_n) be a normalized shrinking basis in E. Then the sequence (e_n^*) of the biorthogonal functionals associated to (e_n) is a Schauder basis of E^*.

(A) Let us first consider the case that K is a compact metric space. According to the preceding proposition, to prove that $C(K, E)$ has the Dieudonné property we must show that every $\sigma(C(K, E)^*, H(C(K, E)))$-convergent sequence in $C(K, E)^*$ is weakly convergent . Let (μ_n) be a sequence in $C(K, E)^*$ which is $\sigma(C(K, E)^*, H(C(K, E)))$-convergent to zero, then, by Lemma 2, $\{|\mu_n|: n \in \mathbf{N}\}$ is uniformly countably additive. It follows from I.2.4 and I.2.5 of [**3**] that there exists a nonnegative measure $\lambda \in \operatorname{rcabv}(\Sigma, \mathbf{R})$ such that

$$\lim_{\lambda(A) \to 0} |\mu_n|(A) = 0 \quad \text{uniformly in } n \in \mathbf{N}. \tag{5}$$

Since E^* is separable, E^* has the Radon-Nikodym property. Hence for each $n \in \mathbf{N}$ there exists $\tilde{\phi}_n \in L_1(\lambda, E^*)$ such that

$$\mu_n(\cdot) = \int_{(\cdot)} \tilde{\phi}_n \, d\lambda. \tag{6}$$

Since (e_n^*) is a Schauder basis of E^*, for every $n \in \mathbf{N}$, there exist $s_n \in \mathbf{N}$ and a simple function $\phi_n\colon K \to E^*$ so that

$$\phi_n(K) \subset \left[(e_j^*)_{j=1}^{s_n}\right] \quad \text{and} \quad \|\phi_n - \tilde{\phi}_n\|_1 < 1/n. \tag{7}$$

Putting

$$\nu_n(\cdot) = \int_{(\cdot)} \phi_n \, d\lambda \quad \text{for } n \in \mathbf{N},$$

it is clear that (ν_n) is $\sigma(C(K, E)^*, H(C(K, E)))$-convergent to zero.

In order to prove that (μ_n) is weakly convergent to zero it is enough to show that (ν_n) is weakly convergent to zero or, equivalently, that (ϕ_n) is weakly convergent to zero in $L_1(\lambda, E^*)$.

Now recall that $L_1(\lambda, E^*)^* \simeq L(L_1(\lambda), E^{**})$ (see VIII.2.2 of [3]). If we suppose that (ϕ_n) is not weakly convergent to zero, then there exist $\tau \in L(L_1(\lambda), E^{**})$, $\varepsilon > 0$ and a subsequence of (ϕ_n) (which we still denote by (ϕ_n)) such that $|\langle \phi_n, \tau \rangle| > \varepsilon$ for all $n \in \mathbf{N}$.

Note that, for every $j \in \mathbf{N}$, the map $\langle e_j^*, \tau(\cdot)\rangle\colon L_1(\lambda) \to \mathbf{K}$ belongs to $L_1(\lambda)^*$; therefore, there exists $g_j \in L_\infty(\lambda)$ so that

$$\langle e_j^*, \tau(f)\rangle = \int_K f(t) g_j(t) \, d\lambda \quad \text{for } f \in L_1(\lambda).$$

It is easily verified that if $\phi\colon K \to E^*$ is a simple function so that $\phi(K) \subset [(e_j)_{j=1}^s]$ for some $s \in \mathbf{N}$, then

$$\langle \phi, \tau \rangle = \int_K \left\langle \sum_{j=1}^{s} g_j(t) e_j, \phi(t) \right\rangle d\lambda.$$

Hence

$$|\langle \phi_n, \tau \rangle| = \left| \int_K \left\langle \sum_{j=1}^{s_n} g_j(t) e_j, \phi_n(t) \right\rangle d\lambda \right| > \varepsilon \quad \text{for } n \in \mathbf{N}. \tag{8}$$

Let $\{P_n\colon n \in \mathbf{N}\}$ be the natural projections associated to the basis (e_n) and let $C = \sup_n \|P_n\|$. If $n, m \in \mathbf{N}$, $n < m$, then

$$\begin{aligned}(P_m^{**} - P_n^{**}) \cdot \tau \in L\left(L_1(\lambda), \left[(e_j^{**})_{j=n+1}^m\right]\right) &\simeq L_1\left(\lambda, \left[(e_j^*)_{j=n+1}^m\right]\right)^* \\ &\simeq L_\infty\left(\lambda, \left[(e_j)_{j=n+1}^m\right]\right);\end{aligned}$$

and since the map $\sum_{j=n+1}^m g_j(\cdot) e_j \in L_\infty(\lambda, [(e_j)_{j=n+1}^m])$ verifies that

$$\langle \phi, (P_m^{**} - P_n^{**}) \cdot \tau \rangle = \int_K \left\langle \sum_{j=n+1}^{m} g_j(t) e_j, \phi(t) \right\rangle d\lambda$$

for $\phi \in L_1(\lambda, [(e_j^*)_{j=n+1}^m])$, one has

$$\left\| \sum_{j=n+1}^{m} g_j(\cdot) e_j \right\|_\infty = \|(P_m^{**} - P_n^{**}) \cdot \tau\| \leqslant \left(\|P_m\| + \|P_n\|\right)\|\tau\| \leqslant 2C\|\tau\|.$$

Up to modifying the functions in a λ-null set if necessary, we can assume that

$$(9)\qquad \left\| \sum_{j=n+1}^{m} g_j(\cdot)e_j \right\|_\infty = \sup_{t\in K} \left\| \sum_{j=n+1}^{m} g_j(t)e_j \right\| \leqslant 2C\|\tau\|$$

for all $n, m \in \mathbf{N}$ with $n < m$.

By (5), (6) and (7)

$$\lim_{\lambda(A)\to 0} \int_A \|\phi_n\|\, d\lambda = 0 \quad \text{uniformly in } n \in \mathbf{N},$$

so there exists $0 < \delta < \lambda(K)$ such that

$$(10)\qquad \int_A \|\phi_n\|\, d\lambda < \frac{\varepsilon}{8C\|\tau\|} \quad \text{for } A \in \Sigma \text{ with } \lambda(A) < \delta, \text{ and } n \in \mathbf{N}.$$

It follows from Lusin's theorem that, for every $j \in \mathbf{N}$, there exists a compact $K_j \subset K$ so that $\lambda(K \setminus K_j) < \delta/2^j$ and $g_{j|K_j}$ (the restriction of g_j to K_j) is continuous. We put $K_0 = \bigcap_{j=1}^\infty K_j$. Then $\lambda(K\setminus K_0) < \delta$; also, since $\delta < \lambda(K)$, $K_0 \neq \varnothing$. Let us denote $h_j = g_{j|K_0}$ for every $j \in \mathbf{N}$.

The series $\sum_{j=1}^\infty h_j(\cdot)e_j$ is weakly Cauchy in $C(K_0, E)$. Indeed, by Theorem 9 of [**4**] it is enough to prove that $\sum_{j=1}^\infty h_j(t)e_j$ is weakly Cauchy in E for each $t \in K_0$. Let $t \in K_0$, $x \in E$ and (p_n) and (q_n) be two increasing sequences in $\mathbf{N}$ so that $p_n < q_n$ for $n \in \mathbf{N}$. By 1.b.1 of [7] the norm of $x^*|\overline{[(e_j)_{j=n}^\infty]}$ (the restriction of x to $\overline{[(e_j)_{j=n}^\infty]}$) tends to 0 as $n \to \infty$, and since

$$\left| \left\langle \sum_{j=p_n}^{q_n} h_j(t)e_j, x^* \right\rangle \right| \leqslant \left\| \sum_{j=p_n}^{q_n} h_j(t)e_j \right\| \left\| x^* \left| \left[\overline{(e_j)_{j=p_n}^\infty} \right] \right. \right\|$$
$$\leqslant 2C\|\tau\| \left\| x^* \left| \left[\overline{(e_j)_{j=p_n}^\infty} \right] \right. \right\|$$

it follows that $\lim_{n\to\infty} \langle \sum_{j=p_n}^{q_n} h_j(t)e_j, x^* \rangle = 0$.

Now, by the Borsuk-Dugundji theorem (see 21.1.4 of [**8**]), there is a bounded linear operator S: $C(K_0, E) \to C(K, E)$, with $\|S\| = 1$, so that:

(a) $S(\phi)(t) = \phi(t)$ for all $t \in K_0$ and for every $\phi \in C(K_0, E)$, and

(b) for each $\phi \in C(K_0, E)$ the values of the function $S(\phi)$ belong to the convex hull of the set $\phi(K_0)$.

Put $f_j = S(h_j(\cdot)e_j)$ for every $j \in \mathbf{N}$. Since the series $\sum_{j=1}^\infty h_j(\cdot)e_j$ is weakly Cauchy in $C(K_0, E)$, then the series $\sum_{j=1}^\infty f_j$ is weakly Cauchy in $C(K, E)$. Therefore there is $\rho \in H(C(K, E))$ such that $\sum_{j=1}^\infty f_j$ is $\sigma(C(K, E)^{**}, C(K, E)^*)$-convergent to ρ.

Note that, by (b), for each $j \in \mathbf{N}$, $f_j(K) \subset [\{e_j\}]$; hence, if $\phi \in L_1(\lambda, E^*)$ and $\phi(K) \subset [(e_j^*)_{j=n}^r]$ for some $r \in \mathbf{N}$, we have

$$\langle \mu, \rho \rangle = \int_K \left\langle \sum_{j=1}^{r} f_j(t), \phi(t) \right\rangle d\lambda$$

where $\mu(\cdot) = \int_{(\cdot)} \phi\, d\lambda$. Therefore

$$\langle \nu_n, \rho \rangle = \int_K \left\langle \sum_{j=1}^{s_n} f_j(t), \phi_n(t) \right\rangle d\lambda \quad \text{for all } n \in \mathbf{N}.$$

On the other hand, according to (8), (9), (10) and (a), for each $n \in \mathbf{N}$ we have

$$\begin{aligned}\varepsilon < |\langle \phi_n, \tau\rangle| &= \left| \int_K \left\langle \sum_{j=1}^{s_n} g_j(t) e_j, \phi_n(t) \right\rangle d\lambda \right| \\ &\leqslant \left| \int_{K_0} \left\langle \sum_{j=1}^{s_n} h_j(t) e_j, \phi_n(t) \right\rangle d\lambda \right| + \left\| \sum_{j=1}^{s_n} g_j(\cdot) e_j \right\|_\infty \int_{K \setminus K_0} \|\phi_n\| d\lambda \\ &< \left| \int_{K_0} \left\langle \sum_{j=1}^{s_n} f_j(t), \phi_n(t) \right\rangle d\lambda \right| + \frac{\varepsilon}{4}.\end{aligned}$$

Thus, for every $n \in \mathbf{N}$,

$$\begin{aligned}|\langle \nu_n, \rho\rangle| &= \left| \int_K \left\langle \sum_{j=1}^{s_n} f_j(t), \phi_n(t) \right\rangle d\lambda \right| \\ &\geqslant \left| \int_{K_0} \left\langle \sum_{j=1}^{s_n} f_j(t), \phi_n(t) \right\rangle d\lambda \right| - \left\| \sum_{j=1}^{s_n} f_j \right\| \int_{K \setminus K_0} \|\phi_n\| d\lambda \\ &\geqslant \frac{3}{4}\varepsilon - \frac{\varepsilon}{4} = \frac{\varepsilon}{2}.\end{aligned}$$

But (ν_n) is $\sigma(C(K, E)^*, H(C(K, E)))$-convergent to zero. This contradiction completes the proof when K is a metric space.

(B) Let K be an arbitrary compact space, let T: $C(K, E) \to F$ be a bounded linear operator which transforms weakly Cauchy sequences into weakly convergent sequences, and let (ϕ_n) be a sequence contained in the unit ball of $C(K, E)$. Similarly as in the proof of Theorem 8 of [1] we can construct a metric compact space $\overline{K}$, a bounded linear operator $\overline{T}$: $C(\overline{K}, E) \to F$ and a sequence $(\overline{\phi}_n)$ in the unit ball of $C(\overline{K}, E)$ such that $\overline{T}(\overline{\phi}_n) = T(\phi_n)$ for all $n \in \mathbf{N}$. Moreover, since T transforms weakly Cauchy sequences into weakly convergent ones, it is immediate that $\overline{T}$ transforms weakly Cauchy sequences into weakly convergent ones, too. By (A), $\overline{T}$ is weakly compact and then $(T(\phi_n))$ has a weakly convergent subsequence. Hence we conclude that $C(K, E)$ has the Dieudonné property.

In view of the above result, the following question arises:

Problem. Does $C(K, E)$ have the Dieudonné property if E does not contain an isomorphic copy of l_1?

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